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# Comparison of LTI and LTP Models for Stability Analysis of Grid Converters

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**Abstract**— The stability analysis of grid-connected converters have attracted increasing attentions, due to the oscillations arising in wind power plants, micro-grids, and other emerging power electronics based power systems. The modeling tool of converters thus becomes essential to faithfully reveal oscillations without any hidden regions. This paper presents a detailed comparison of two linearized modeling methods, which are, respectively, developed in the Linear Time-Invariant (LTI) and the Linear Time-Periodic (LTP) frameworks. The LTP model can consider the effect of frequency-coupling dynamics, which are occurred by the time-varying behavior, while the conventional LTI model can not capture this behavior. The advantages and limits of two models are then illustrated with examples. The compared results are verified in the frequency domain and time domain as well.

**Keywords**—LTI system, LTP system, stability analysis, Harmonic State Space Modeling, power converter

## I. INTRODUCTION

The linearization of power converters has always been an important topic over the past few decades. In recent years, the widespread use of voltage-source converters in electric railway networks [1], renewable power plants, and microgrids [2] tend to bring in the unexpected harmonics and even unstable oscillations at the higher frequencies, ranging from hundreds of hertz to kHz [1]. These resonance and instability phenomena are mainly due to the control and switching interactions between converters and passive components in a converter-dominated power system.

However, it is recently found that the resonance is not only shifted through the time-varying behavior but also generated at the unexpected frequency due to the frequency coupling from the multiplication of several time-varying signals. Hence, the main challenge lies in the time-varying behavior of power converters, and the differences of the reported modeling methods in the field of power electronics are how to appropriately treat the time-varying dynamics for the specific problems [1].

The importance of including the time-varying behavior in the model has also been reported in several applications, where the unexpected harmonics and instabilities are manifested due to the frequency-coupling dynamics of the converter [1], [3]–[5]. The low-order harmonic frequency oscillations have been found in the thyristor-based static VAR compensators and high-voltage direct current systems [3], [4]. It is partly due to the resonant behavior of the ac system, where the low order harmonics may be magnified by the high

impedance of the ac system, and the magnified low-order harmonics may in turn cause the unstable operations of converters. Hence, the linearized modeling technique that takes the time-varying effect into account is increasingly demanded for grid-converters.

Several modeling methods have been developed to deal with the time-varying effect in converter models. Generalized averaging (also known as dynamic phasor) methods [6] and the multi-frequency averaging technique were developed to include the influence of the switching-frequency harmonics [5]. The harmonic linearization model was recently introduced to improve accuracy by including the frequency-coupling effect, which is driven by the time-varying modulation in the model [7]. It is noted that the effective frequency-coupling is taken by the modulation (SPWM, SVPWM, etc) [8]. Furthermore, the Harmonic Domain (HD) based models (also known as the Harmonic State-Space) were introduced to linearize all the time-varying components of the power converter [9], [10]. It is noted that even though the modeling methods have been proposed with slightly different names and concepts, they can be classified into two categories as LTI and LTP model. The main factor of classification is whether the model includes the time-varying behavior or not, and how they treat them in the modeling procedure if they have. However, these models are complex to understand regarding the behavior and the limitation, which may prevent the models from being properly used in the analysis.

This paper presents the model differences of LTI and LTP models for the stability analysis of power converter. Firstly, the main component, which makes the time-varying effect, is deeply discussed. Additionally, the structures of two models are explained by introducing the principal modeling methods, which have been researched in the field of power electronics. Furthermore, the difficulties of including the time-varying behaviors are explained with reasons why they are necessary in order to achieve an accurate model. Secondly, the frequency responses of two models are compared with the time-domain simulation. Furthermore, a comparison of the models used for the stability analysis is also presented to elaborate the limitation of each model and to know which model cannot effectively predict the stability.

## II. STRUCTURE AND STABILITY ANALYSIS OF LTI AND LTP MODEL

The power converter has a formulation (1) in the model, where all parameters and signals are nonlinear time-varying.

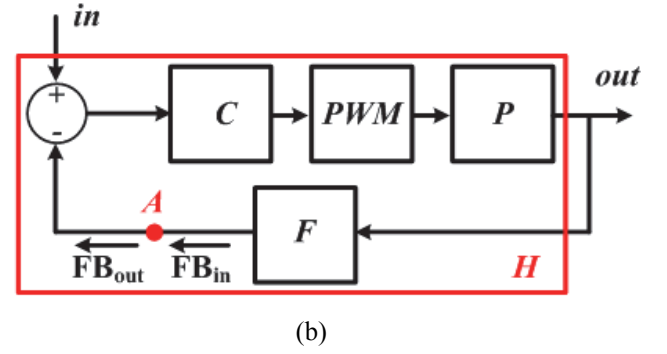
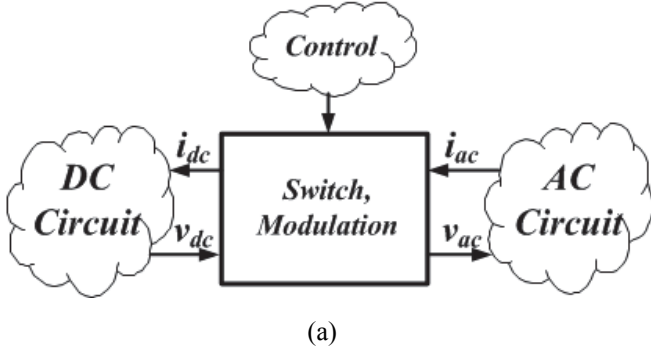


Fig. 1. (a) General structure of Power converter, (b) Generalized block diagram of power converter, where “C” is controller, “PWM” is modulation and time delay, “P” is plant (filter) and “F” is feedback gain.

However, (1) should be linearized in order to analyze the stability using small-signal analysis. The linearization method can be classified into three categories. For instance, linearization at the specific operating point, linearization according to the trajectories and linearization according to the periodic trajectories. In this paper, the first and third cases are only considered to compare the LTI and LTP model.

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned} \quad (1)$$

Furthermore, this section is composed of two parts, where the structure of LTI and LTP model will be described in advance based on (1) and how it can be converted into two models. Additionally, the structure of the conventional modeling is also explained with detailed discussions on how to know the range of their coverage in the modeling procedure. Secondly, two ways of stability analysis in the LTI and the LTP system will be described based on the first part.

#### A. Structure of LTI and LTP model

The model given in (1) can be converted into two general structures as given in (2) and (3) through the linearization [9], [11].

$$H_{ltp_k}(s) = \sum_l \hat{C}_{k-l} \left( (s + j\omega_0)I - \hat{A} \right)^{-1} \hat{B}_l + D_k \quad (2)$$

$$H_{lti}(s) = C(s - A)^{-1}B + D \quad (3)$$

where,  $\hat{B}_k$ ,  $\hat{C}_k$ , and  $\hat{D}_k$ , are the Fourier coefficients of the periodic functions  $B(t)$ ,  $C(t)$ , and  $D(t)$ , respectively.  $H_{ltp_k}(s)$  is the transfer function of LTP system and  $H_{lti}(s)$  is the transfer function of LTI system. Additionally,  $H_k(s)$  is a double infinite matrix defined in (5), where the matrix defines the coupling between different frequencies. On the contrary to the LTP model, the LTI model has only single transfer function ( $H_{lti}(s)$ ), but the LTI transfer function can also have the formulation (4), where (4) shows the input frequency having a fundamental frequency ( $\omega$ ) which will only react with the shifted LTI transfer function. This property is also the reason why power converters can use Proportional Resonant (PR) controller to control the fundamental component. Hence, it is worth to note that the matrix (4) is a LTI model, even if it has an infinite input and output.

Furthermore, one of the remarkable characteristics of the LTP system is that an LTP system can also be defined as an LTI system with infinitely Multiple Inputs and Multiple Outputs (MIMO) through the linearization according to the periodical trajectories. The difference between (4) and (5) is the coupling behavior and its effectiveness to other closed transfer function.

$$H_{lti}(s) = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots \\ \dots & H_0(s - j\omega) & 0 & 0 & \dots \\ \dots & 0 & H_0(s) & 0 & \dots \\ \dots & 0 & 0 & H_0(s + j\omega) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (4)$$

$$H_{ltp}(s) = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \vdots \\ \dots & H_0(s - j\omega) & H_{-1}(s) & H_{-2}(s + j\omega) & \dots \\ \dots & H_1(s - j\omega) & H_0(s) & H_{-1}(s + j\omega) & \dots \\ \dots & H_2(s - j\omega) & H_1(s) & H_0(s + j\omega) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (5)$$

In the model structure of (4) and (5), a representative modeling can be defined. Firstly, the “traditional average modeling” has only  $H_0(s)$  [11] and it shows Single Input and Single Output (SISO) characteristic. It is noted that traditional modeling does not care about the ac information. The well-known “dq-reference frame modeling” and “multi-frequency averaging” [12], [13] are considering  $H_0(s + j\omega)$  based on the assumption where the fundamental frequency governs the whole stability, and the negative frequency may not have any influence due to the characteristic of the conjugate. The results show that the transfer function of the d-q axis has the same property with the real part of  $H_0(s + j\omega)$  ( $= \text{Re}[H_0(s + j\omega)]$ ) and the imaginary part of  $H_0(s + j\omega)$  ( $= \text{Im}[H_0(s + j\omega)]$ ). Additionally, two impedances are coupled where the case is to find the coupling the dq-axis. [14] discusses the influence of the negative frequency in the dq-system by using the complex transfer function when unsymmetrical systems analyzed. They define two complex transfer functions as the positive frequency part ( $\text{Re}[H_0(s + j\omega)]$ ,  $\text{Im}[H_0(s + j\omega)]$ ) and the negative frequency part ( $\text{Re}[H_0(s - j\omega)]$ ,  $\text{Im}[H_0(s - j\omega)]$ ), and analyze the unsymmetrical systems mainly based on the fundamental frequency. It is noted that the dq-model is a part of the LTP model and the coupling is only between the real and the imaginary part of the fundamental frequency.

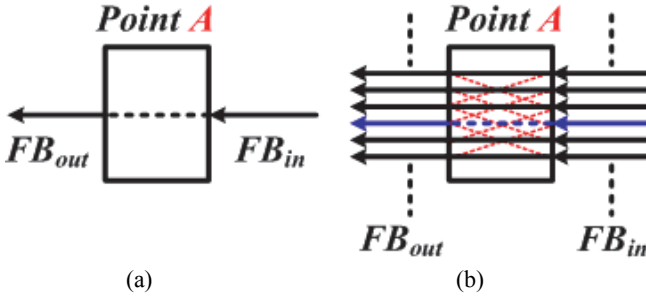


Fig. 2. Perturbation point in LTI and LTP model of Fig. 1-(b), (a) LTI view of “point A”, (b) LTP view of “point A”

Furthermore, the crucial part of the dq-model is that it is the same as the frequency shifting in the averaged model.

On the contrary to both introduced cases, the “generalized averaging modeling (=dynamic phasor)” [6] take  $H_1(s)$  as well as  $H_{-1}(s)$  in the model procedure to consider the effect of the negative frequency. Additionally, the generalized average modeling use in a describing function at the end of result to linearize the nonlinear equation, which means the coupled behavior  $H_{-1}(s) \dots H_1(s)$  are only considered in the modeling result according to the assumption where the most significant signal is the fundamental components.

Similarly, the “harmonic linearization” method [7] can be explained by (5) where it takes one transfer function at the end of the result by using a describing function and harmonic balance technique. It is worth to note that both the generalized averaging and the harmonic linearization methods are considering the coupled response at the end. However, the describing function and harmonic balance approaches enforce the single input to single output map, afterwards higher harmonics can be neglected and it leads to inaccurate results [15]. Furthermore, the possibility of another coupling at the different frequency could be neglected, and it can bring less accurate result than considering more frequency coupling.

However, another approach, the “Harmonic Domain (HD) based method” [10], [16], which is also called the Harmonic State Space (HSS) modeling, is recently introduced to consider the whole behavior of the time-varying components in the system by using the structure of a frequency response as given in (5). It can include the harmonic components as much as the user wants to include in the modeling, and it can be used for the analysis of other frequency responses in the system. The structure itself is a canonical model of the LTP system [9], and a similar structure is introduced in the field of power system and power electronics using the name “Multi-frequency averaging” [17] or “Multivariable dq-reference frame” [12]. Though the structure of them seems similar, the difference is in the way they treat the real and imaginary part of the multi-frequency response as well as their information is averaged or not.

As a result, the introduced principal methods can be classified into the LTI and LTP model. The LTI model is a special case of the LTP model, and the dq-model, the generalized averaging model, and the harmonic linearization are a part of the LTP model or the reduced order model of

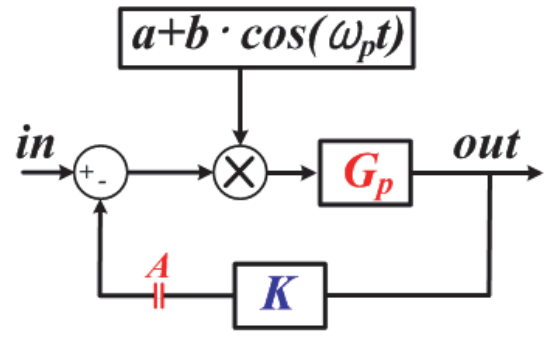


Fig. 3. Canonical LTP model from Fig. 1-(b), where the effect of PWM is regarded as “cos(.)” function

LTP system. Furthermore, the multi-frequency, multivariable model, and the HD-based model can be regarded as the full framework of the LTP model, which are considering the coupling behavior with other frequency components as given (5). Even though the importance of a full model is crucial to consider all behaviors of coupling inside the system, (5) is more complex than the (4), and it makes difficult to analyze the system due to the coupled impedance as given in (5).

The time-varying component is crucial in ac-dc or ac-ac systems, and it should be included in the modeling procedure as well as in the analysis. Hence, at least, the dq-model, which is considering the positive frequency part of the fundamental component, should be used in the analysis. A simplified averaging model can not be used properly to investigate the time-varying behavior in a system. The HD-based model has the possibility to show the whole behavior of the system depending on the particular case to be studied.

#### B. Stability analysis of LTI and LTP model

When doing small-signal stability analysis using a LTI and LTP system, the perturbation procedures for frequency response should be concerned by different approaches, where Fig. 1-(b) can be regarded as the LTI system or the LTP system.

As already known in the traditional analysis, a single frequency should be injected with a small magnitude in order to achieve the frequency response. The result drawn by the perturbation is a response with respect to the perturbed frequency. As a result, the model can be used for the stability analysis at the single (dc) operating point. The LTI system is stable if the contour of LTI Nyquist plot does not encircle (-1,0) in the analysis. For instance, the traditional average model can be judged by using a SISO-LTI Nyquist plot. The detailed behavior of LTI feedback loop is shown in Fig. 2-(a) that it is required to cut the feedback loop (A) in Fig. 1-(b) to draw the Nyquist plot of the open-loop transfer function. In the case of Fig. 2-(a), the user can only perturb the single-frequency to the input ( $FB_{in}$ ) and the resulting response ( $FB_{out}$ ) is the responding single frequency as well. Hence, only a single Nyquist-plot can be drawn.

The stability analysis in the reduced order LTP systems (the dq-reference model, the generalized averaging model, the harmonic linearization) can be performed by using the

generalized Nyquist criterion for MIMO (Multi Input Multi Output) systems [18]. If the open loop system is stable, the Nyquist contours of the eigenvalues do not enclose the point “(-1,0)”, then the closed loop system is stable. The perturbation of the LTP model should be performed by a different approach compared to the LTI model due to the coupling. The point A is composed of a MIMO system, which is coupled as shown in Fig. 2- (b) and the number of an input and output depends on the size of the truncated matrix (5). Hence, the user should disconnect one connection like the “blue-line” in Fig.2-(b). If the user just perturbs one frequency (“blue line”) of input ( $FB_{in}$ ) as shown in Fig.2-(b), the response ( $FB_{out}$ ) of Fig.2-(b) will show the coupled output, and it gives the stability for the frequency of interest.

In contrary to the reduced order LTP model, the Nyquist plot of the full-order LTP model can also be drawn by using a different approach. According to the introduced theory in [19], the frequency range of  $\omega$  for the contour varies from  $\omega = -\omega_p/2$  to  $\omega = +\omega_p/2$ , then the Nyquist diagram forms a CCW (Counter Clock Wise) circuit. Instead of the normal Nyquist plot, the LTP inverse Nyquist diagram can be plotted for interpretation. If the feedback gain is defined as “k”, the encirclements of the  $-k$  point will be counted instead of the  $-1/k$  point in the normal Nyquist diagram [19]. For all  $k>0$ , the response on the negative real axis, the closed loop system is stable. For all  $k<0$ , the responses of the positive real axis, the closed-loop system is unstable. The interpretation of the inverse Nyquist plot is the same as the analysis of the traditional Nyquist diagram.

### III. SIMULATION COMPARISON OF LTP MODEL

Matlab is used as a tool to compare the LTI and LTP model at the same conditions. Firstly, the canonical LTP model “*Lossy Mathieu equation*” [9] is considered in the simulation in order to give inspiration to the differences between the LTI and LTP model. Secondly, a single-phase grid-connected converter is modeled and simulated according to the introduced LTP theory.

#### A. LTP stability in “*Lossy Mathieu equation*.”

The canonical model of the LTP system is taken into account as given in (6) to explain the simple behavior of LTP and LTI model, where (6) can be the LTI model when  $b$  is equal to “0”. Additionally, (6) is straightforward a 2<sup>nd</sup> order differential equation and the important factors are “a, b,  $\omega_p$ ” which make the periodic time-varying behavior in the model. It is noted that (6) can simply be regarded as the modulated converter system, where the behavior of the PWM (modulation) in Fig. 1-(b) can be regarded as the time-varying behavior ( $a + 2b\cos(\omega_p t)$ ) of LTP model in (6). Furthermore, a cosine function, which is rotating with the same frequency as “ $\omega_p$ ”, is considered as the input of (6) and the overall behavior of (6) is the same with the modulated single-phase system.

$$\ddot{x}(t) + 2\zeta\dot{x}(t) + (a + 2b\cos(\omega_p t))x(t) = I\cos(\omega t) \quad (6)$$

Equation (6) can be modeled in Matlab as shown in Fig. 3 to draw the open-loop Nyquist plot after cutting off a point

“A”. The plant “ $G_p$ ” of (6) is  $1/(s(s + 2\zeta))$  and  $\zeta$  is assumed to “0.2” in this case. The magnitude of input is assumed to be “1”. The “2” is assumed to be the period of  $\omega_p$ , where the frequency is “ $1/\pi$ ”. The “b” and “K” are mainly varied to investigate the effect of the time-varying element in (6) and to know how the LTP Nyquist plot can show the difference with the LTI Nyquist plot.

The result of the comparison between the two models is shown in Fig. 4. LTI and LTP models show the same frequency response (Nyquist plot) in Fig. 4-(a) and (b) even if the system in Fig. 3 is modeled by using the LTP theory. The internal properties of the LTP structure may have the same structure with (4) because the time-periodic signal modulates any input. Thus, the time-domain simulation results are also exactly the same when “b” is equal to “0” as shown in Fig. 4-(c). The two models show stable operation, and the “-K” point is inside of the contour, which means the system is stable according to the definition of the inverse Nyquist plot. The difference appears as “b” is increasing. The Nyquist plot, which is derived from the LTP model, has a symmetric ripple because of time-varying behavior as shown in Fig. 4-(d). It is worth to note that considering the time-varying component shows different frequency responses with the LTI model, where it has the same frequency response with Fig. 4-(a) even if “b” is changed. Furthermore, the stable region starts to be separated and reduced compared to the Fig. 4-(b). Additionally, the different waveforms, which are shown in the LTP model of Fig. 4-(e), have a dc-offset because of the time-varying behavior. However, the time response from the two models still shows a stable operation because “-K” is still in the stable region. The feedback gain “K” is adjusted to “1.2” in order to verify the stable region of Fig. 4-(d). The magnified part of Fig. 4-(d) is shown in Fig. 4-(f). “-1” is still inside of the contour and it means that the system is stable. However, the time-domain simulations of the LTP model is exponentially diverging as shown in Fig. 4-(g) when “K” is adjusted to “1.2”, where “-K (= -1.2)” is outside the Nyquist contour as shown in Fig. 4-(f). Compared to the LTP model, the LTI model still shows a stable behavior in the frequency domain as well as in the time domain because “-K (= -1.2)” is still inside the stable contour.

The Nyquist contour of the LTP model starts to draw a circle as “b” is increasing as shown in Fig. 4-(h), where the stable region is more reduced and separated. The “-K” is out of the circle contour in Fig. 4-(h) and it gives the unstable behavior of the time-domain simulation as well as shown in Fig. 4-(i). However, the LTI model still shows the same response with the other case. The simulation results from Fig. 4-(d) to (i) show exactly how the stable region can vary according to the magnitude of the varying signal, which can not be taken into account in the LTI model.

#### B. LTP stability in a “*Single-phase grid-connected converter*”

Based on the knowledge from the stability analysis of the canonical model, a power electronic based system is analyzed by using the same theory. The single-phase grid-connected converter is considered as an example to investigate the effect of time varying modulation as well as their influence on the

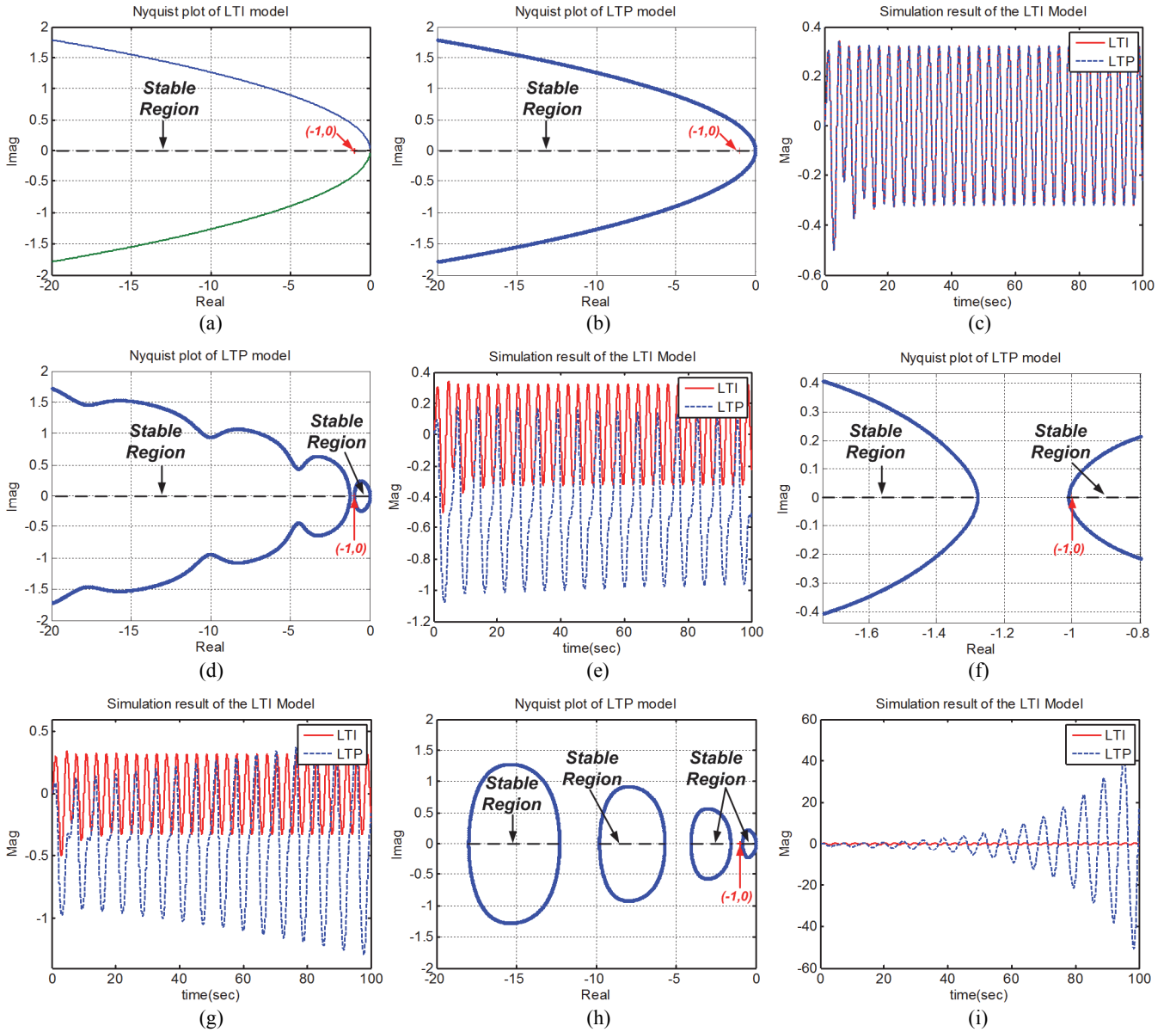


Fig. 4. Comparison of LTI and LTP model (a) Nyquist plot (@ Point A of Fig. 3) of LTI model, (b) Nyquist plot (@ Point A) of LTP model, when  $a=1 / b=0$ , (c) Time-domain simulation of LTI and LTP model, when  $a=1 / b=0$ , (d) Nyquist plot (@ Point A) of LTP model, when  $a=1 / b=0.4 / K=1$ , (e) Time-domain simulation of LTI and LTP model, when  $a=1 / b=0.4 / K=1$ , (f) Magnified Nyquist plot (@ Point A) of LTP model, when  $a=1 / b=0.4 / K=1$ , (g) Time-domain simulation of LTI and LTP model, when  $a=1 / b=0.5 / K=1$ , (h) Nyquist plot (@ Point A) of LTP model, when  $a=1 / b=0.5 / K=1$ , (i) Time-domain simulation of LTI and LTP model, when  $a=1 / b=0.5 / K=1$

stability region. A block diagram of a single-phase grid-connected converter is shown in Fig. 5, where a single inductor ( $L_f, R_f$ ) is considered as the input ac-filter. The converter is controlled in rectifier mode by controlling the dc-voltage of the capacitance ( $C_{dc}$ ) and dc-load ( $R_{dc}$ ). The grid impedance is assumed as a simple  $L$ - $C$  circuit ( $L_g, R_g, C_f$ ). A simple controller is also used that a sensed dc-voltage ( $v_{dc-sense}$ ) through low-pass filter ( $H_{LPF}$ ) is controlled by PI controller. Then, the current reference ( $i_f^*$ ), which is derived from the output of PI controller ( $|i_f^*|$ ) and PLL ( $\cos_{PLL}(\cdot)$ ), is adjusted by the PR-controller. The output of the current controller (PR) is compared with a saw-tooth waveform to

generate the switching sequence. The PLL is not taken into account in the LTP modeling for simplicity and it is regarded as a synchronized cosine function with grid voltage ( $v_g$ ).

The block diagram in Fig. 5-(a) is modeled by using the HSS theory, which is also based on the LTP theory [20]. It is noted that the modulation signal, as well as the switching behavior, are the main time-varying components in the modeling procedure, but other components can also be modeled as time-varying components. Furthermore, all signals are harmonic vectors, and each component has a matrix formulation, where the final structure has a formulation of a MIMO-LTI system. Based on the HSS modeling results, Fig.



5-(a) can be depicted as shown Fig. 6-(b) to represent the impedance interaction between the converter impedance / admittance ( $P(s)$ ) and the grid impedance / admittance ( $L(s)$ ). All s-domain functions in Fig. 5-(b) represent a matrix, where  $Q(s)$  means the matrix between the dc-voltage reference ( $\Delta v_{dc}^*$ ) and ac-current ( $\Delta i_f$ ) and  $K(s)$  means the matrix between the PCC-voltage vector ( $\Delta v_{pcc}$ ) and the grid-voltage vector ( $\Delta v_g$ ).

The loop gain ( $P(s)/L(s)$ ) is used to analyze the stability of the system when “K” is equal to “0”. The main time-varying signal of the power converter systems is a modulation signal, and it is typically varying according to the variation of the reference in the controller. Hence, the modulation index is varied in order to investigate the effect of the varying behavior as explained in the simple example of the LTP system in Fig. 3 and Fig. 4. Two cases are considered in the analysis, where 1 kW and 0.1 kW are considered to emulate the varying behavior of the modulation signal. Their behavior in the time domain simulation is shown in Fig. 7-(a) and the power rating is changed at 0.4 sec from 1 kW and 0.1 kW. The controller for harmonic compensation is not considered in ac as well as dc-circuit and they include odd order harmonics in ac-current and even order harmonic in dc voltage. The contents of harmonics are changed due to the modulation procedure, when the power rating is changed as shown in Fig. 7-(a). However, even if the power rating is changed, the system is stable as shown in Fig. 6-(a) and (b), where  $-10^{\text{th}} \sim 10^{\text{th}}$  order harmonics are considered in the LTP Nyquist plot. The different colors in Fig. 6-(a) and (b) mean the contours, which are drawn by the different harmonic transfer functions according to the LTP theory. It is noted that the stable and unstable regions (black line) are changed when the reference of the power rating is changed at 0.4 sec, where Fig. 6-(a) and (b) are obtained at the different linearization points. Furthermore, Fig. 6-(b) shows different circle sizes and varied contours that reflects the behavior of the time-varying element. The basic theory is the same as the example in Fig. 4 that it is same with varying “b” value in Fig. 3.

The unstable case is also analyzed when the “K” value is increased to “30”. Two Nyquist plots are compared for the different conditions as shown in Fig. 6-(c) and (d). Fig. 6-(c) shows the contour when “K” is equal to “1”. The “(-1,0)” point is still in the stable region and the analyzed results match with the time-domain simulation results in Fig. 7-(b) from 0 sec to 0.4 sec. However, the system starts to become unstable when the factor “K” is “30”. The linearized LTP Nyquist plot is shown in Fig. 6-(d), where “(-1, 0)” is in the unstable region and it brings the unstable behavior in the time domain simulation as shown in Fig. 7-(b). The contour is governed by the properties of each harmonic transfer function, and it changes the stable region.

#### C. Discussion regarding the importance of harmonics

It is investigated in “section-B” that considering the time-varying components in the model is critical for the stability analysis since the stable and unstable regions are changing according to the magnitude of the varying elements. Furthermore, the used numbers of harmonics transfer functions in the model are related to the accuracy of model

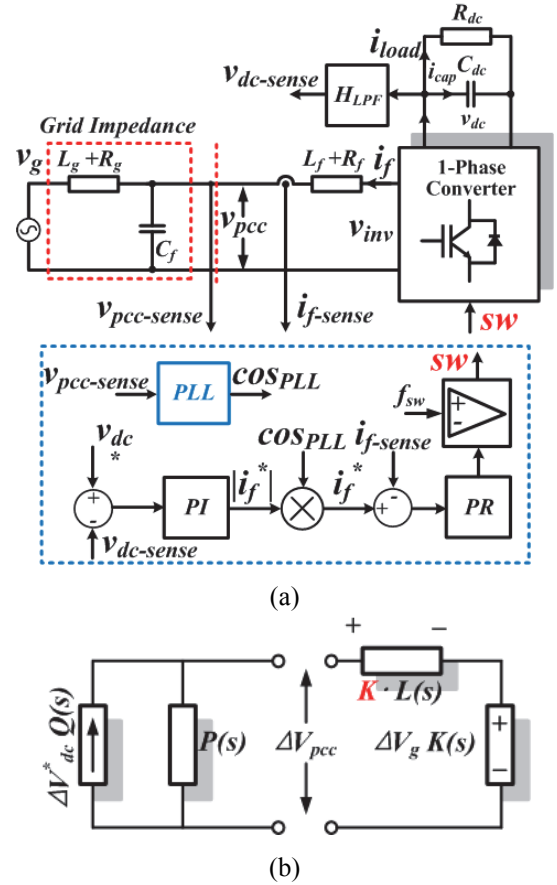


Fig. 5. Single-phase grid-connected converter with grid impedance (a) Block diagram of topology and control system, (b) Impedance representation of single-phase grid-connected converter

properties. It is worth to note that the results in Fig. 6 are partially similar to a dq-domain model, which is considering the unbalanced system, or the generalized averaging model, which is considering the fundamental frequency as the largest component. As a conclusion, the reduced LTP model (dq-model, generalized averaging) can take into account the effect of varying modulation with the duty value. Additionally, the variation of stability regions according to the considered harmonics could be neglected for the model reduction. However, the reduced LTP model has an error and small differences compared to the full LTP model (considering at least  $10^{\text{th}}$  order harmonics). The simulated case in this paper is a simple example, and the influence of harmonics can be more critical, if there are other varying elements in the model or the power converter operates at the low frequency ( $<10$  Hz). Additionally, the importance of other harmonics can be more important applications like diode or thyristor converters, which are having a grid-dependent switching.

#### IV. CONCLUSION

This paper presents the nature of the LTI and LTP model for the analysis of power converters. The main criteria and difficulties in the modeling procedure are introduced including that the original behavior of a power converter is a nonlinear

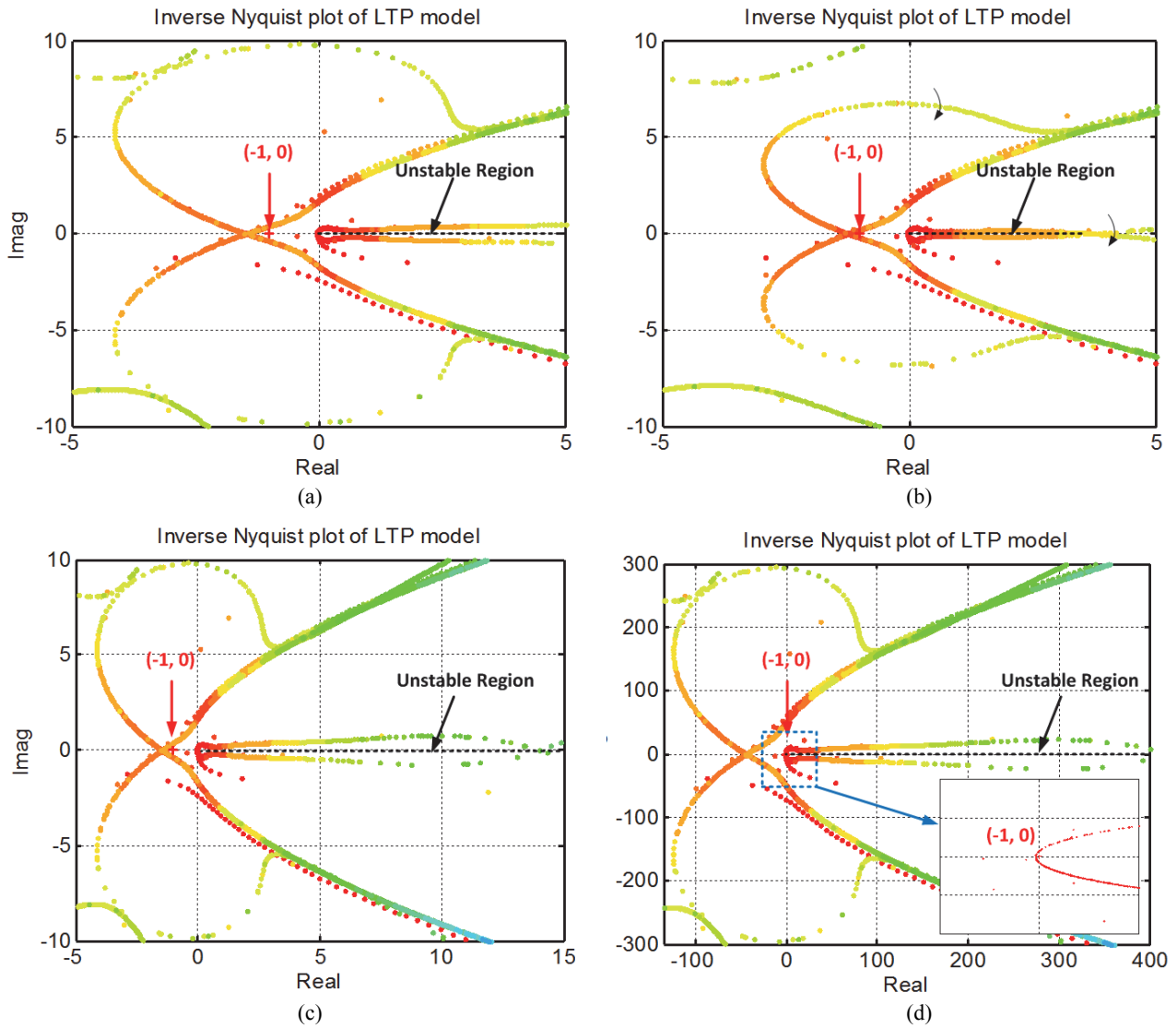


Fig. 6. Nyquist plot of Loop gain ( $P(s)/L(s)$ ) of single phase grid connected converter (a) LTP Nyquist plot of Loop gain (1 kW), (b) LTP Nyquist plot of Loop gain (0.1 kW), (c) LTP Nyquist plot of Loop gain (1 kW,  $K=1$ ), (d) LTP Nyquist plot of Loop gain (1 kW,  $K=30$ )

time-periodic system and also how the difficulties can be covered through the LTI and LTP theory. A simple model is adapted explicitly to explain the differences in the frequency domain and time domain for the two modeling methods. The results show that the analysis based on the LTP theory is more acceptable for power converters, as the time-varying behavior can not be considered in the LTI model.

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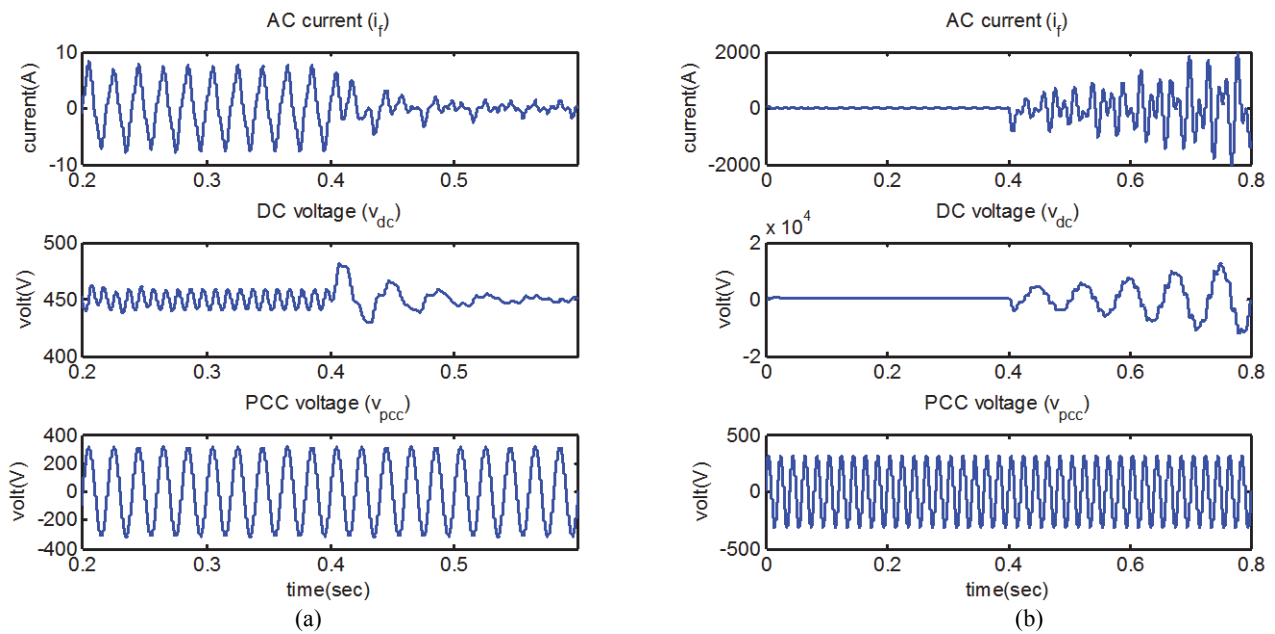


Fig. 7. Time domain simulation results of Fig. 5 (a) Power variation from 1 kW to 0.1 kW at 0.4 sec, (b) Grid impedance variation at 0.4 sec

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